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Math 10/11 Honours: Section 7.4 Shortest Distance Points and Lines

1. Given each line, find the coordinates of the "x" and "y" intercepts:

a) $y=-\frac{4}{5}x+11$

y-intercept:

11

x-intercept:

$\frac{55}{4}$

b) $7x-8y=-28$

y-intercept:

$\frac{7}{2}$

x-intercept:

-4

c) $9y-3x+21=0$

y-intercept:

$\frac{21}{9}$

x-intercept:

7

2. Given each line, find the shortest distance from the origin (0,0):

a) $y=-\frac{3}{4}x+8$

$3x+4y-8=0$

$(x,y)=(0,0)$

$d=\frac{|4\cdot 0+3\cdot 0-8|}{\sqrt{4^2+3^2}}=\frac{8}{5}$

b) $8x+6y=24$

$8x+6y-24=0$

$d=\frac{|8\times 0+6\times 0-24|}{\sqrt{8^2+6^2}}=\frac{24}{10}=\frac{12}{5}$

c) $9y+4x+36=0$

$d=\frac{|4\cdot 0+9\cdot 0+36|}{\sqrt{9^2+4^2}}$

$d=\frac{36}{\sqrt{97}}$

3. Determine the shortest distance from each point to the line:

a) $y=\frac{2}{3}x+8 \quad (-6,11)$

$2x-3y+8=0$

$d=\frac{|2(-6)-3(11)+8|}{\sqrt{2^2+3^2}}=\frac{37}{\sqrt{13}}$

b) $3x+5y=15 \quad (-10,3)$

$d=\frac{|3(-10)+5(3)-15|}{\sqrt{3^2+5^2}}$

$d=\frac{30}{\sqrt{34}}$

c) $3x+4y-28=0 \quad (7,6)$

$d=\frac{|3(7)+4(6)-28|}{\sqrt{3^2+4^2}}=\boxed{\frac{17}{5}}$

4. Determine the distance between each pair of parallel line:

a) $3x+5y=10$ contains point $(0,2)$
$3x+5y=3$

$d=\frac{|3(0)+5(2)-3|}{3^2+5^2}$

$d=\frac{7}{\sqrt{34}}$

b) $y=\frac{2}{3}x+4$ contains point $(0,4)$
$y=\frac{2}{3}x-7 \Rightarrow 2x-3y-7=0$

$d=\frac{|2(0)-3(4)-7|}{\sqrt{2^2+3^2}}=\frac{19}{\sqrt{13}}$

c) $3x-4y+12=0$ $(0,3)$
$3x-4y+18=0$

$d=\frac{|3(0)-4(3)+18|}{\sqrt{3^2+4^2}}=\frac{6}{5}$

5. Given the line equation L_1: $3x+4y+2=0$ is a tangent to the circle "C" centered at (-3,-2). Find the equation of the circle.

$\text{radius}=\frac{|3(-3)+4(-2)+2|}{\sqrt{3^2+4^2}}=\frac{15}{5}=3$

eqn: $(x+3)^2+(y+2)^2=9$

B) Find the equation of the other tangent line to the circle that is parallel with L_1

$3x+4y+2=0$

$\text{radius}=3=\frac{|3(-3)+4(-2)+p|}{\sqrt{3^2+4^2}} \Rightarrow 15=|-17+p|$

$\cancel{p_1=2}$ $p_2=20$
we already have this

$\boxed{3x+4y+20=0}$

6. A line through B(0,-10) is 8 units from the origin. Determine its equation:

$y=m(x+10) \Rightarrow y-mx-10m=0$ $d=8$ $(0,0)$

$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}} \Rightarrow 8=\frac{|1(0)-m(0)-10m|}{\sqrt{1+m^2}} \Rightarrow 8\sqrt{1+m^2}=10m$

$1+m^2=\frac{25}{16}m^2 \Rightarrow \frac{9}{16}m^2=1 \Rightarrow m^2=\frac{16}{9}$

$m=\pm\frac{4}{3}$

$\boxed{\text{equation: } y=\pm\frac{4}{3}x\pm\frac{40}{3}}$

7. Two lines with a slope of 2, L_1 passes through (-3,1) and L_2 passes through (9,0). Find the distance between L_1 and L_2

$y-1=2(x+3) \Rightarrow y-2x-7=0 \Rightarrow 2x-y+7=0$

$y=2(x-9)$ through $(9,0)$

$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}=\frac{|2(9)-1(0)+7|}{\sqrt{2^2+1^2}}=\frac{25}{\sqrt{5}}=\boxed{5\sqrt{5}}$

8. Given the line equation L_k: $kx - y - k - 1 = 0$, where "k" can be all real numbers, what point must all the lines pass through?

$-y-1 = k - kx$

$y+1 = kx - k$

$y+1 = k(x-1)$

slope $= k$

point $= (1,-1)$

$(1,-1)$

9. Given the circle equation: $x^2 + y^2 - 6x - 8y - 24 = 0$ and line equation: $4x + 3y + C = 0$. For what values of "C" will the circle and line intersect at two different points?

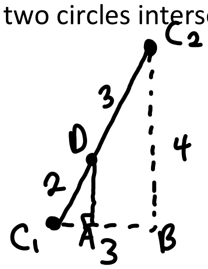
$0 = x^2 + y^2 - 6x - 8y - 24 \Rightarrow (x-3)^2 + (y-4)^2 = 49$

if tangent, the line has to be 7 units away from $(h,k) = (3,4)$.

$7 = \frac{|4(3) + 3(4) + C|}{\sqrt{4^2+3^2}} \Rightarrow 35 = \pm(24+C) \Rightarrow C = 11, -59$

$\boxed{-59 \leq C \leq 11}$

10. Given two circles C1 and C2, with centers (-1,2) and (2,6) respectively, and radius 2 and 3 respectively. If the two circles intersect at only one point, find the point of intersection.



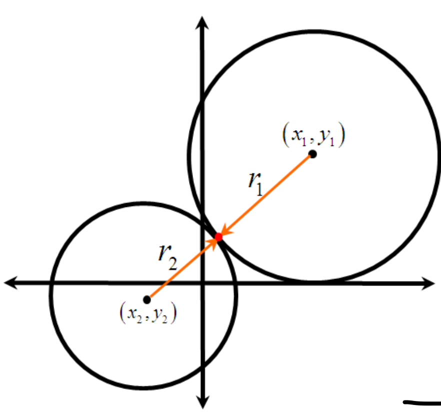
$\Delta C_1AD \sim \Delta C_1BC_2$

$\frac{C_1A}{C_1B} = \frac{C_1D}{C_1C_2} \Rightarrow \frac{C_1A}{3} = \frac{2}{5} \Rightarrow C_1A = \frac{6}{5}$

By the same principle, $AD = \frac{8}{3}$ $\frac{8}{8}$

$\boxed{D\left(\frac{1}{5}, \frac{18}{5}\right)}$

11. Challenge: Given circle C_1 with center (x_1, y_1) and radius r_1, and circle C_2 with center (x_2, y_2) and radius r_2 and that they intersect at only one point, prove that the point of intersection (x_m, y_m) is given by the formula: $x_m = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2}$ and $y_m = \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2}$



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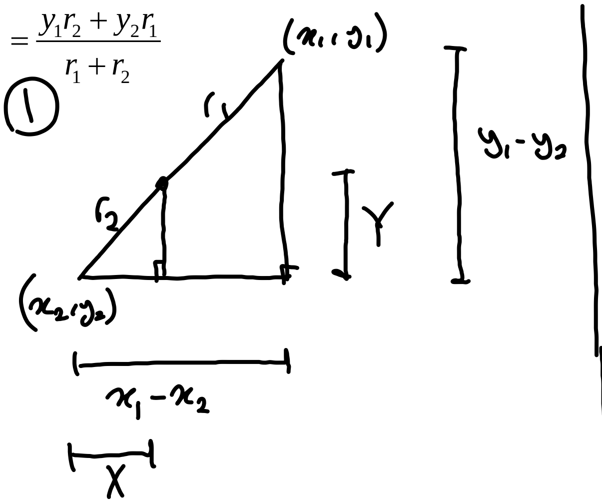

② $\frac{r_2}{r_1 + r_2} = \frac{X}{x_1 - x_2} \Rightarrow X = \frac{r_2(x_1 - x_2)}{r_1 + r_2}$

x-coordinate: $\frac{r_1(x_1 - x_2)}{r_1 + r_2} + x_2$

③ $\frac{r_2}{r_1 + r_2} = \frac{Y}{y_1 - y_2}$

$Y = \frac{r_2(y_1 - y_2)}{r_1 + r_2}$

y-coord: $\frac{r_2(y_1 - y_2)}{r_1 + r_2} + y_2$

④ $(x, y) = \left[\frac{r_2(x_1 - x_2) + x_2(r_1 + r_2)}{r_1 + r_2}, \frac{r_2(y_1 - y_2) + y_2(r_1 + r_2)}{r_1 + r_2}\right] = \boxed{\left(\frac{x_1 r_2 + r_1 x_2}{r_1 + r_2}, \frac{r_2 y_1 + y_2 r_1}{r_1 + r_2}\right)}$

12. Super Challenge: The formula for the shortest distance "D" between a point P$\left(x_1, y_1\right)$ and a line $Ax + By + C = 0$ is given by the formula $D = \frac{\left|Ax_1 + By_1 + C\right|}{\sqrt{A^2 + B^2}}$. The proof for this formula can be very challenging. Use this page to prove this formula.

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